DYNAMIC MODELING, DESIGN AND CONTROL OF WIRE-BORNE UNDERACTUATED BRACHIATING ROBOTS: THEORY AND APPLICATION

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DYNAMIC MODELING, DESIGN AND CONTROL OF WIRE-BORNE UNDERACTUATED BRACHIATING ROBOTS: THEORY AND APPLICATION

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LIST OF ABBREVIATIONS

ADC analog-to-digital converter

AFA adaptive function approximation

ASMC adaptive sliding mode control

CB control bound

CBF control barrier function

CLF control Lyapunov function

DOF degrees of freedom

EOM equations of motion

FL feedback linearization

GPIO general-purpose input/output

IMU inertial measurement unit

IR infrared

LQR linear quadratic regulator

MCU microcontroller unit

NLP nonlinear programming

ODE ordinary differential equation

PDE partial differential equation

PID proportional-integral-derivative

PWM pulse width modulation

PWMNC pointwise min-norm control

QP quadratic programming

QP-AFA-RCLBF quadratic programming with adaptive function approximation and robust control Lyapunov and control barrier function

RCBF robust control barrier function

RCLBF robust control Lyapunov and control barrier function

RCLF robust control Lyapunov function

RES-CLF rapidly exponentially stabilizing control Lyapunov function

RK4 fourth-order Runge-Kutta

RMS root mean square

RMSE root mean square error

RRT rapidly-exploring random trees

SDP semidefinite programming

SOS sum-of-squares

TVLQR time-varying linear quadratic regulator

UART universal asynchronous receiver/transmitter

SUMMARY

The ability of mobile robots to locomote safely in unstructured environments will be a cornerstone of robotics of the future. Introducing robots into fully unstructured environments is known to be a notoriously difficult problem in the robotics field. As a result, many of today's mobile robots are confined to prepared level surfaces in laboratory settings or relatively controlled environments only. One avenue for deploying mobile robots into unstructured settings is to utilize elevated wire networks. The research conducted under this thesis lays the groundwork for developing a new class of wire-borne underactuated robots that employs brachiation – swinging like an ape – as a means of locomotion on flexible cables.

Executing safe brachiation maneuvers with a cable-suspended underactuated robot is a challenging problem due to the complications induced by the cable dynamics and vibrations. This thesis studies, from concept through experiments, the dynamic modeling techniques and control algorithms for wire-borne underactuated brachiating robots, to develop advanced locomotion strategies that enable the robots to perform energy-efficient and robust brachiation motions on flexible cables. High-fidelity and approximate dynamic models are derived for the robot-cable system, which provide the ability to model the interactions between the cable and the robot and to include the flexible cable dynamics in the control design. An optimal trajectory generation framework is presented in which the flexible cable dynamics are explicitly accounted for when designing the optimal swing trajectories. By employing a variety of control-theoretic methods such as robust and adaptive estimation, control Lyapunov and barrier functions, semidefinite programming and sum-of-squares optimization, a set of closed-loop control algorithms are proposed. A novel hardware brachiating robot design and embodiment are presented, which incorporate unique mechanical design features and provide a reliable testbed for experimental validation of the wire-borne underactuated brachiating robots. Extensive simulation results and hardware experiments demonstrate that the proposed multi-body dynamic models, trajectory optimization frameworks, and feedback control algorithms prove highly useful in real world settings and achieve reliable brachiation performance in the presence of uncertainties, disturbances, actuator limits and safety constraints.

CHAPTER 1

INTRODUCTION AND BACKGROUND

Brachiation is a form of swinging [1, 2] used efficiently by primates and other mammals to locomote within unstructured environments which contain networks of elevated support structures, such as tree canopies. Akin to walking, brachiation is adaptable to a dynamic environment, e.g., non-uniformly spaced and oriented handholds, possibly interspersed with obstacles, and likely prone to vibrations and other disturbances.

As the demand for automation and services provided by robotic systems grows, there is an increasing need to be able to deploy mobile robots into *unstructured environments* such as farm fields [3], urban areas [4], and forests [5]. Reliable locomotion in these types of environments has, to date, been difficult to achieve without the use of complex multi-legged robots [6, 7, 8]. In fact, in unstructured environments ranging from cities to farmland, the ability of mobile robots to locomote in a robust manner independent of human intervention is at once both extremely important, and extremely challenging. Wheeled robots suffer from a number of known limitations and cannot traverse obstacles larger than about the size of the wheel radius. Likewise, the current state-of-the-art in legged robots is not sufficiently advanced for them to be deployed in an autonomous fashion in outdoor settings. Aerial robots and UAVs, while able to operate free from ground obstructions, have limited flight time (typically on the order of tens of minutes) due to high power consumption and are

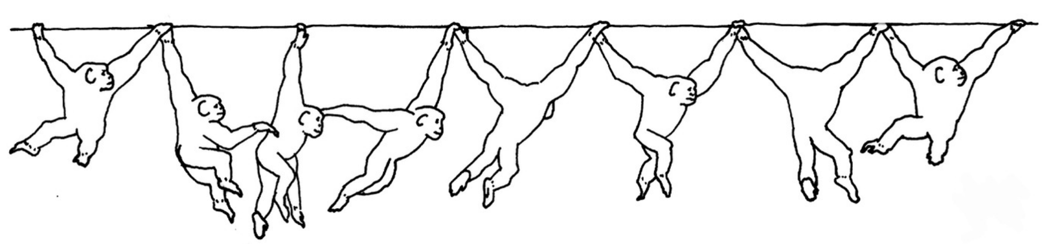


Figure 1.1: A gibbon performing brachiation maneuver [2]. Reproduced with permission of The Licensor through PLSclear.

potentially dangerous to operate. As a result, many of today's mobile robots are confined to prepared level surfaces in controlled environments only.

One avenue of deployment for mobile robots into unstructured settings is to utilize elevated wire networks, either pre-existing or installed specifically to enable robot locomotion. Many operating environments, from urban areas to pastures, are equipped with networks of elevated wires that, in addition to their original purpose, can also serve as a locomotive infrastructure for mobile robots. For instance, cities are equipped with power or telephone lines. Vineyards are equipped with wires that support plant growth. Indeed, preexisting wire networks are ubiquitous in many environments that can benefit from the presence of mobile robots, but that otherwise may be inaccessible due to locomotion issues. Even in cases where wire networks do not already exist, they can oftentimes be easily installed. By leveraging these wires for locomotion, a host of new environments are opened up where mobile robots can "live" and serve in a variety of important roles. As an example, consider a farm field in which a wire is installed over each crop row, or a city with pre-existing power lines. A robot capable of traversing the wire network will thus be able to access areas of interest without needing to physically interact with complex obstacles.

1.1 Motivation

One attractive aspect of brachiation, compared to other locomotion techniques such as legged locomotion, is that rather than avoiding obstacles, brachiating robots attempt to leverage obstacles as support structures to enable mobility. In fact, brachiation can be viewed as a generalized version of walking in which the contacts with the ground surface or obstacles are adhesive, i.e., the feet of a walking robot become grippers. This generalization allows brachiating robots to be deployed to a wide range of environments, as long as this adhesion or gripping capability can be effectively implemented. The magnetic foot brachiating robot reported by Mazumdar and Asada [9], which is designed to walk inverted below steel bridges, exemplifies this notion of brachiating robots as generalized walking robots.

Despite the fact that the past several decades has seen increasing interest in the use of brachiation as a locomotion modality for mobile robots [10, 11], they have not yet emerged in real life scenarios. A major challenge in deploying brachiating robots in real life applications comes from the uncertainties and disturbances present in outdoor environments. Moreover, in the current literature, brachiating robots have been researched almost exclusively for rigid bars/supports [12, 13]. However, this has limited applicability in real-world situations since many environments may not be easily configurable to accommodate rigid structures. By contrast, wire traversing robots [14, 15] have a better chance of getting deployed in real life applications, as it is relatively easier to install a flexible wire or cable in outdoor environments.

There is great potential for *wire-traversing* robots to be adopted in real life settings, ranging from smart cities applications such as monitoring and surveillance, traffic management, and public safety, to industrial domains such as power line inspection and precision agriculture. Any task that involves traversal of unstructured environments, or coexistence with humans in a shared space, is potentially well-suited for execution by wire-borne robots since they offer predictable, reliable locomotion and significant mission flexibility. Wire-borne robots essentially offer a mechanism to introduce mobile robots into unstructured environments, provided that a cable network is available for locomotion. Fortunately, pre-existing wire networks are ubiquitous (or could potentially be installed) in many environments that may benefit from the presence of mobile robots, but that otherwise may be inaccessible due to locomotion issues. For instance, existing power transmission lines or the overhead wires for trolley/bus systems in urban areas may be leveraged as a medium for locomotion by wire-borne robots that serve as reconfigurable sensor networks for adaptive surveillance or traffic management within cities. Likewise, wire-borne robots could be used for persistent plant health monitoring in precision agricultural applications. Such mobile robots may also play a role in wildlife monitoring and protection. Many wildlife sanctuaries contain vast arrays of wire networks meant originally for containment, but which also could be easily used for the proposed robots. In addition to the broad range of applications mentioned above, there are clear applications of this technology to numerous other domains including conservation, security, consumer technology, and even home healthcare.

Examples of wire-traversing robots have recently emerged as *rolling* [15, 16] and *brachiating* [14] robots. Brachiating robots offer unique advantages over robots that simply roll along a wire in that they have the capability to bypass obstacles (e.g. tree branches or aerial marker balls) as well as to swing to adjacent cables. However, the vibrations in the flexible wire induced by locomotion or external disturbances such as winds may significantly complicate the control of the robot. Moreover, as brachiating robots move off rigid bars and out of laboratories, the notion of *safety* will come to attention, as the robot would need to operate in a safe region to avoid collisions with obstacles in the environment.

When considering controller design and modeling and simulation of such systems, several challenges arise. First, the *dynamics* of the coupled robot-cable system are complex and high-order. Second, measuring or *estimating* the states of the cable during a robot swing is mostly infeasible due to sensing limitations. Finally, *robustifying* the control design to enable the robot to locomote on such a vibrating medium is challenging due to the wide variety of initial conditions that may be encountered and the inherent uncertainty in the system dynamics. This is especially important in real-world settings where safe and reliable performance of the robot must be guaranteed. Developing solutions to these challenges will be an important step in transitioning brachiation robots from their current status as curious laboratory demos to practical robots that can solve real world problems.

Considering the challenges described above, the ultimate goal of this thesis is to design and control a new type of robust, energy efficient, and low-cost brachiating robot, that can be cost-effectively deployed in situ and provide tangible benefits to real world systems with different applications. Note that a key aspect of our vision for these wire-borne robots is that they can be made small, low-power, autonomous, and to a certain extent inconspicuous and symbiotic with their surroundings. The development and real world implementation of new locomotion and control strategies for such brachiating robots will be instrumental in eventually bridging the gap between research and real world applications, and achieving desired requirements of an automated system: low-cost and scalable, capable of near-persistent operation, and capable of operating robustly in a real world environment.

1.2 Problem Formulation

Figure 1.2: The wire-borne underactuated brachiating robot hardware prototype.

The present thesis has as its objectives the dynamic modeling, robotic system design, motion planning, feedback control synthesis, and experimental validation of a wire-borne underactuated brachiating robot, a mobile robot that moves using its arms similar to an ape moving from branch to branch, by swinging like a pendulum attached to a flexible cable. The robot cannot apply torque at its grip, since it has an unactuated joint on the pivot arm. Therefore, this type of robot is categorized as an underactuated system by having fewer actuators than degrees of freedom.

The work presented here involves a strong coupling between dynamic modeling, control theory, mechanical design, simulation analysis and experimental testing. Wire-traversing brachiating robots pose interesting mobility challenges, especially when engineering constraints of robustness and low-cost are considered. Such robots fall into a broader class of highly-dynamic robotics: underactuated mobile robots which exploit gravity and momentum and involve contact with the environment. We study the kinematics, dynamics and control methods for brachiating robots to develop advanced locomotion strategies, which will permit the robot to perform energy-efficient and robust brachiation motions, in the presence of unmodeled dynamics, estimation uncertainties, safety constraints, and actuator limits.

An optimal control strategy is developed to derive desired optimal reference trajectories for brachiation motions. The dynamical model is improved by including the flexible cable model in the design in order to capture the behavior of the system and its response with respect to the cable motion/vibration, which enhances control of the robot's dynamic locomotion. Four feedback control algorithms, namely a parameterized time-varying linear quadratic regulator (TVLQR), a robust sum-of-squares (SOS) optimization-based, an adaptive sliding mode control (ASMC), and a robust and adaptive quadratic programming (QP)-based control framework, are synthesized for the task of underactuated, torque limited brachiation on flexible cables, in order to identify the exact feedback control laws necessary to generate reliable and robust brachiating maneuvers, enabling wire-borne brachiating robots to locomote safely in a robust manner without human intervention and in the presence of constraints, uncertainties and disturbances caused by flexible cables and obstacles. The resultant control motions and performances are characterized and compared via an extensive Monte Carlo analysis with respect to various evaluation metrics, including energy, accuracy and feasibility.

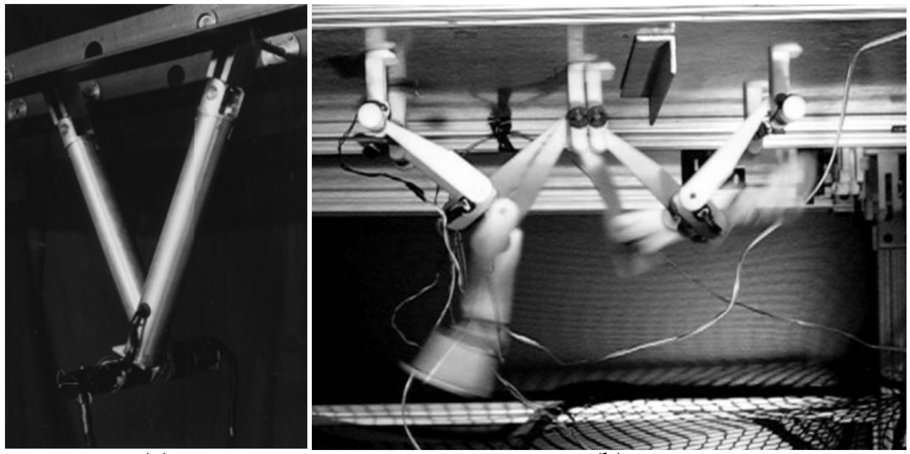
A mechanical design and the associated mechatronic systems are presented for brachiating robots traversing flexible cables, which will seek to balance the reliable wire-brachiating capability with size and power restrictions. The novel wire-traversing mobile robot to be developed and fabricated in this work is designed to attach to, and traverse, elevated wires, and provides a suitable testbed for experimental validation of the proposed control frameworks in this thesis. The proposed robot hardware is shown in Figure 1.2. The robotic system envisioned here is comprised of a two-link brachiating robot with a single actuator situated at the joint between the robot's two arms, and two active grippers at two ends of the robot which perform necessary grasping for brachiation. The robot is intended for use with flexible cables, which represents a departure from the literature in which robot brachiation has been reported almost exclusively for relatively rigid rods/supports. To achieve a cost-effective solution, not only does the brachiating robot itself need to be low-cost but the cable infrastructure it inhabits must be as well. To this end, we employ a simple extra flexible cable, elevated and held up by only two supports at each end.

In this work, simulation studies are used to predict and evaluate the performance of the proposed dynamic models and control methods. Once performance is deemed satisfactory in simulation, the proposed control and locomotion algorithms are further validated in an experimental setting by conducting extensive real-world experiments on the brachiating robot prototype traversing a flexible cable.

1.3 Related Work

Over the past decade, a variety of wire-borne robots has been designed that leverages wheels for locomotion. Nearly all of these robots have been developed solely for the purpose of powerline inspection: the robots are designed to roll along miles of elevated cable and identify areas of the transmission line that require maintenance. Some examples include the transmission line inspection robot built by the Electric Power Research Institute [17], the Cable Crawler robot developed at ETH Zurich [18], and the HiBot Expliner robot [19]. Because these robots roll along the wire (rather than brachiating), bypassing an obstacle poses a major challenge for these systems. Upon encountering a powerline support, these robots typically execute a series of slow, choreographed maneuvers in a quasi-static fashion to disconnect from one wire, circumvent the obstacle, and attach on the opposite side. Such complex maneuvers typically require large numbers of actuators and sensors and can take many minutes to complete. As a result, these powerline inspection robots have tended to be relatively expensive and slow moving. These limitations are generally acceptable for powerline inspection missions but are less desirable for other use cases such as urban and agricultural monitoring. For the most part, previous attempts to design wire-traversing robots have resulted in designs that are large, somewhat cumbersome, and very expensive – the kinds of robots that are designed to be built in quantities of one or two. In contrast, the mobile robot envisioned in this thesis, which will regularly traverse a simple flexible wire and will operate in and amongst humans, will need to be fundamentally different from these heavy, expensive, and cumbersome machines.

The field of brachiating robots has also been explored extensively over the past two decades. However, the research efforts on control of brachiating robots have mainly focused on brachiation on rigid structures, such as ladders and monkey bars. The concept of a brachiating robot was first introduced by Fukuda [20, 21] as a new type of mobile robot that could make use of the efficient swing motion of a pendulum due to gravity to locomote on ladder bars (shown in Figure 1.3(a)). Later, Saito *et al.* [22, 10, 23] proposed a heuristic control algorithm by repetitive trial and error, followed by a reinforcement learning implementation [24], for a two-degree-of-freedom robot swinging on horizontal parallel bars. The Target Dynamics algorithm was proposed in [12] to enable continuous locomotion of a simplified two-link brachiating robot over several rungs of a ladder. Using this method, instead of handling the system dynamics via reference trajectories, the control task is achieved by representing the robot dynamics with a simplified single pendulum as a lower dimensional target. However, the controller requires knowledge of the exact dynamics of the robot. Spong in [25] and [26] proposed a partial feedback linearization method for the swing-up control of the "Acrobot", a two-link underactuated robot with a similar mechanism to brachiating robots. Zero-energy cost motions for passive brachiating models attached to a rigid ceiling were investigated in [27], proposing mathematical solutions



(a) (b)

Figure 1.3: Examples of brachiating robots in the literature: (a) brachiation robot by Fukuda [10], (b) Mag-Foot robot by Mazumdar [9] designed for bridge inspection.

that do not demand any joint torque at any time. Mazumdar and Asada [9, 11] designed an underactuated brachiating robot with magnetic "feet" for steel bridge inspection, which incorporates passive magnets for attachment to the bridge structure, and uses a feedback linearization-based controller to track optimal motion trajectories. Their brachiating robot, called "Mag-Foot", is shown in Figure 1.3(b). Similarly, Gibbot [28], a brachiating robot to locomote vertical walls, utilized electromagnets hands and an open-loop control method to perform "downhill" and "uphill" brachiation gaits. In [29], a PD control and an adaptive robust control were employed to derive energy-minimizing swing trajectories and track optimal trajectories for a two link brachiating robot with uncertain kinematic and dynamic parameters moving between fixed supports, showing a 25% energy reduction compared to the target dynamics method proposed in [12]. An optimal control framework to exploit passive dynamics of a two-link brachiating robot with a variable stiffness actuation mechanism was presented in [30]. Model predictive control has also been explored in the context of brachiation, both in a nonlinear [31] and linearized form [32]. A model-free sliding mode control scheme was presented in [33, 34] to control symmetric and asymmetric robotic brachiators along a rigid bar with an upward slope.

An interesting mechanical design was proposed in [35] to address the wire-traversing problem through brachiation, demonstrating different locomotion modalities along a flexible wire. However, its methods of locomotion provide no feedback control used for traversing the cable, and therefore prone to failure for real-world applications. More recently, a three-link brachiation robot was presented in [13], which used an iterative linear quadratic regulator (LQR) algorithm for trajectory generation and a combination of a cascaded proportional-integral-derivative (PID) control and an input-output linearization controller to track desired trajectories and swing along monkey bars. Each of these methods has drawbacks that include some combination of long training periods, infeasibility of real-time implementation, strong performance dependency on initial conditions, not being robust against uncertainties and disturbances, or control chattering phenomena [36] that can be detrimental to actuators.

As is evident in the literature review above, the problem of brachiating robot control on a flexible medium has been mostly neglected in the literature, due to the uncertainties and challenges created by introducing a flexible body into the system. Modifying the swing trajectory of a brachiating robot to avoid obstacles and at the same time achieve control objectives is another challenge that has likewise not been well-studied.

When considering the problem of brachiation on flexible structures, common approaches for control of fully-actuated or underactuated nonlinear dynamical systems can be investigated to control a wire-borne brachiating robot. These control schemes can be roughly divided into the categories of optimal control problems, classical state and output feedback regulators, and control Lyapunov-based approaches, some of them employing adaptive and robust methodologies to deal with unknown and uncertain conditions. We review the relevant control algorithms that can be leveraged to control an underactuated brachiating robot over a flexible cable in the following.

With regard to optimal control problems, one common approach in the literature is to use a library of trajectories to design controllers for constrained nonlinear systems [37, 38, 39, 40]. Tedrake et al. [38, 41] presented the LQR-trees algorithm, which builds a sparse tree of LQR-stabilized trajectories [42, 43] and verifies the regions of attraction using Lyapunov functions. Liu and Atkeson [44] developed a balance controller for a two-link robot based on a trajectory library and dynamic programming to generate local linear approximations to an optimal trajectory. A rapidly-exploring random trees (RRT) framework was used in [45] to plan feasible trajectories for nonlinear dynamical systems including the Acrobot. However, these methods are often computationally expensive, cannot be implemented in real time, and lack the robustness to uncertainties and disturbances in the model or environment.

Recent developments in semidefinite programming (SDP) and sum-of-squares (SOS) optimization [46, 47] have resulted in development of Lyapunov-based state-feedback controllers along with formal guarantees of their region of attraction (for time-invariant systems), or their invariant sets (for time-varying systems) via sum-of-squares programming [48, 49, 50, 51, 52], which can accommodate external disturbances and model uncertainties in the dynamics. These approaches can be mainly categorized into two methods: synthesizing closed-loop controllers while minimizing the outer approximation of the reachable sets [52], versus feedback control design by maximizing the inner approximation of the backward reachable sets [48, 53]. While the former method is better suited for real-time planning in unknown environments, the latter provides the advantage of driving to a predefined goal from a larger set of initial conditions using a single reference trajectory.

In addition to the control methods above which employ sum-of-squares programming, effective classical approaches in the controls literature for robust control of underactuated systems include sliding mode control [54, 55, 56], adaptive control [57, 58, 59, 60] and backstepping [61, 62]. Robust controllers can be employed to mitigate the effects of modeling errors and bounded disturbances on a system's stability and performance. Sliding mode control [63, 64] is an efficient robust control method that has been widely used to control systems with bounded disturbances and uncertainties [65, 66], entailing construction of a surface onto which the error asymptotically converges to zero. However, designing a stable sliding manifold is not straightforward for underactuated systems [67]. Moreover, to tune the constant gains of robust control terms, the bounds of modelling errors and disturbances need to be known in advance, which is not the case for many applications. Direct adaptive methods [68, 69] can be applied to form a time-varying control gain and automatically compensate for bounded disturbances without the need to know the bounds a priori. Using a direct adaptive design, instead of identifying the unknown system parameters, the gains of the control law are directly adjusted by an adaptive update law without any intermediate calculation so that the desired tracking performance is achieved. Indirect adaptive methods are particularly common in the robotics control literature [70, 71, 72], where the adaptive law generates on-line estimates of the unknown parameters of the system dynamics which then are used to calculate the control law. However, to guarantee parameter convergence and achieve zero error tracking, adaptive methods rely on the reference trajectory to be persistently exciting [73], which is not always ensured for dynamical systems. Additionally, the performance of adaptive controllers may be significantly degraded or even lead to instability if disturbances and unmodeled dynamics are too large in the system.

Nonetheless, none of the classical control approaches reviewed above can handle the presence of unsafe (obstacle) regions, and each of them has drawbacks that include some combination of strong performance dependency on initial conditions and/or control chattering phenomena that can be detrimental to actuators.

In recent years, a large body of literature has been created studying control Lyapunov function (CLF)-based controllers [74], which leverage online quadratic programs (QPs) to incorporate additional constraints including stability, input-based, and state-dependent constraints into the control computation [75, 76]. In [77] and [78], it was shown that CLF conditions and additional constraints can be unified into a single QP framework and solved online. Rapidly exponentially stabilizing control Lyapunov functions (RES-CLF) were introduced in [75], which can guarantee exponential stability of periodic orbits in hybrid systems with a controlled convergence rate. A main assumption in the formulation of CLF-QP controllers is that the full dynamic model of the system is known. Robust and Adaptive CLFs [79] were proposed in [80] and [81] to handle model uncertainty via quadratic programs for nonlinear hybrid systems such as bipedal walking robots.

To include safety-critical constraints in the control design, control barrier functions (CBFs) [82, 76] are utilized which convert safety constraints into linear inequality constraints that can be incorporated into quadratic programs. Exponential control barrier functions are developed in [83] to expand the use of CBFs for constraints with relative-degree higher than 1. Barrier states (BaS) are introduced in [84], which, when embedded in a control system's model, can avoid the conflict between control objectives and safety constraints in a QP-based control design. Nevertheless, the development of CLFs and CBFs in the domain of brachiating robots has not been investigated to date.

While an extensive body of brachiation control research has been established, a key missing element in prior work is treatment of a flexible support in the context of modeling and control, which is a major focus of this thesis as will be detailed in Section 1.4. To the best of our knowledge, none of the prior works in the literature has addressed the problem of brachiating on a vibrating medium, nor can handle the uncertainties and challenges introduced by a flexible cable in a system.

1.4 Structure of Thesis and Contributions

While a strong foundational knowledge base underpins the field of brachiating robots, the work proposed here aims to advance the state-of-the-art in this domain so that brachiating robots become viable mobile robots for real world applications.

The structure of this work proceeds as follows. In Chapter 2, we present the development of high-fidelity and approximate multi-body dynamic models to formulate deterministic and stochastic equations of motion for underactuated brachiating robots attached to flexible cables. The proposed methods enable the underlying control systems to capture the dynamic effects of the cable on the robot and account for the relative vibrations and collisions taking place between the robot's grippers and the cable. The approximate dynamics models enable inclusion of parametric model uncertainties in the system.

Development of an optimal trajectory generation framework for the robot-cable system is presented in Chapter 3. A major advantage offered by the trajectory framework lies in the treatment of a flexible support in the context of optimal control. The proposed parametric trajectory optimization approach reduces the computational complexity in the nonlinear optimization program, and enables the resulting framework to employ the high-fidelity dynamic model and explicitly accounts for the flexible cable dynamics when generating optimal swing trajectories.

A closed-loop stabilizing controller based on the time-varying LQR algorithm is presented in Chapter 4. The resulting feedback control framework is used in conjunction with the feedforward optimal trajectory, in order to correct for disturbances and drive the system toward its desired final states. The control is made robust to time-delays and perturbation by incorporating a variable look-ahead scheme implemented by reparameterizing the trajectory in terms of system states rather than time.

In Chapter 5, building on the work on optimal trajectory generation and TVLQR controller presented in the previous chapters, we use semidefinite programming (SDP) and sum-of-squares (SOS) optimization to synthesize a time-varying feedback control with formal robustness guarantees to account for model uncertainties and unmeasurable states in the system. Simulations and comparison with the TVLQR controller demonstrates that the proposed robust controller results in relatively large robust backward reachable sets in the presence of parametric model uncertainties, actuator limits, and unobservable states. The proposed design leads to the first SOS-based robust controller design in the domain of underactuated brachiating robots.

A novel estimation-based approach to model the interactions between the flexible cable dynamics and the robot without using any sensors is presented in Chapter 6. Moreover, the formulation of a combined direct-indirect adaptive sliding mode control (ASMC) scheme for wire-borne underactuated brachiating robots in the presence of parametric uncertainties and unmodeled dynamics is presented, along with formal stability analysis and adaptation law derivations for the proposed control design using a Lyapunov stability argument. The superiority of the proposed controller over the widely used input-output feedback linearization method for underactuated systems is presented through simulation experiments. The proposed design leads to an adaptive robust control framework that compensates for the unknown cable force without knowing the bound of discrepancy between the approximated and actual force a priori, enabling underactuated brachiating robots to traverse flexible cables in an online fashion.

In Chapter 7, robust control Lyapunov and barrier functions are designed and incorporated into quadratic programs to synthesize a unified adaptive QP control framework for the wire-borne brachiating robot. The proposed control design formally guarantees the stability and safety of the robot in the presence of dynamic uncertainties, actuator constraints and obstacles in the environment. Stability analysis and derivation of adaptation laws are carried out through a Lyapunov analysis. Simulation results and comparisons with a baseline controller show that the proposed quadratic programming-based controller achieves reliable tracking performance and disturbance estimation in the presence of unstructured uncertainties, actuator limits and safety constraints.

Design and fabrication of a novel and scalable mechanical brachiating robot, including a novel gripper design, distributed embedded systems and software architecture development is presented in the first part of Chapter 8. The proposed robot design presents a breakthrough improvement in brachiating robot technology that is a prerequisite for practical use in real-world applications.

In the second part of Chapter 8, we perform hardware experimental validation for the proposed feedback control strategies by conducting real-world experiments on the robot prototype traversing a flexible cable. The experimental results provide the first hardware evaluation of locomotion techniques for wire-borne underactuated brachiating robots in an experimental setting.

Finally, in Chapter 9, we conduct an extensive Monte Carlo analysis to characterize and compare the performance of the proposed control algorithms with regard to different evaluation metrics. An exploration of trade-offs between the proposed control schemes will inform critical decision making for selecting planning and control strategies needed to ensure reliable locomotion of wire-borne brachiating robots in real-world applications.

The conclusions of this work and future directions are addressed in Chapter 10. The proposed multi-body dynamic models, trajectory optimization frameworks, and feedback control algorithms presented in this thesis may prove highly useful in real world applications in which brachiating robots must traverse elevated wires, tree limbs, or other non-rigid support structures.

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