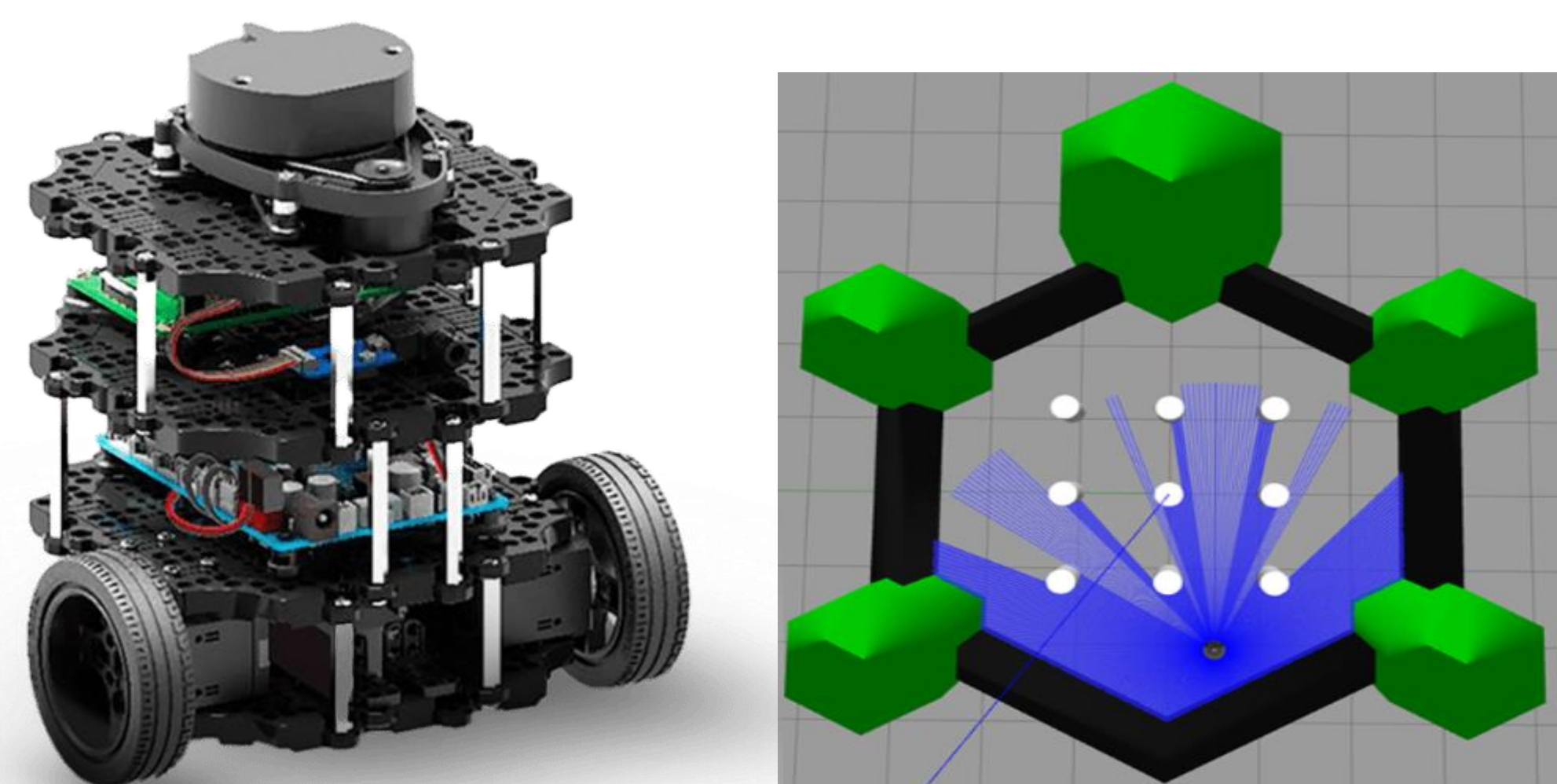


# Localization and Formation Control For Multi-Agent Systems

Dmitri Dobrynin, Indigo Garcia  
Advisor: Dr. Siavash Farzan  
Electrical Engineering Department, California Polytechnic State University, San Luis Obispo, CA



## Localization

### Motivation

- GPS insufficient for multi-agent systems
- Localization algorithms use relative pose measurements
- Need a solution for measurement noise and consensus between agents

### Background

- Undirected graph of n nodes (agents)
- Sensing model for measurements:  
 $z(k) = H(k)x(k) + v(k)$
- Target with discrete time model dynamics:  
 $x(k+1) = Ax(k) + Bw(k)$
- Noise,  $v$  and  $w$ , follow zero-mean white Gaussian noise
- Discrete time implementation of a Kalman consensus filter [1]
- Local data aggregation:

$$y_i = \sum_{j \in J_i} H_j^T R_j^{-1} z_j \quad S_i = \sum_{j \in J_i} H_j^T R_j^{-1} H_j$$

- Kalman-Consensus estimate:

$$M_i = (P_i^{-1} + S_i)^{-1}$$

$$\hat{x}_i = \bar{x}_i + M_i(y_i - S_i \bar{x}_i) + \epsilon M_i \sum_{j \in N_i} (\bar{x}_j - \bar{x}_i)$$

- Update state:

$$P_i \leftarrow A M_i A^T + B Q B^T$$

$$\bar{x}_i \leftarrow A \hat{x}_i$$

### Methodology

- Distributed Kalman filtering algorithm [1]
  - Filters measurement noise
  - Agents determine own pose
- Python simulation of mobile agents
- To be addressed in future work:
  - Increase number of agents
  - Time-varying adjacency matrices
  - Vary agent measurement noise

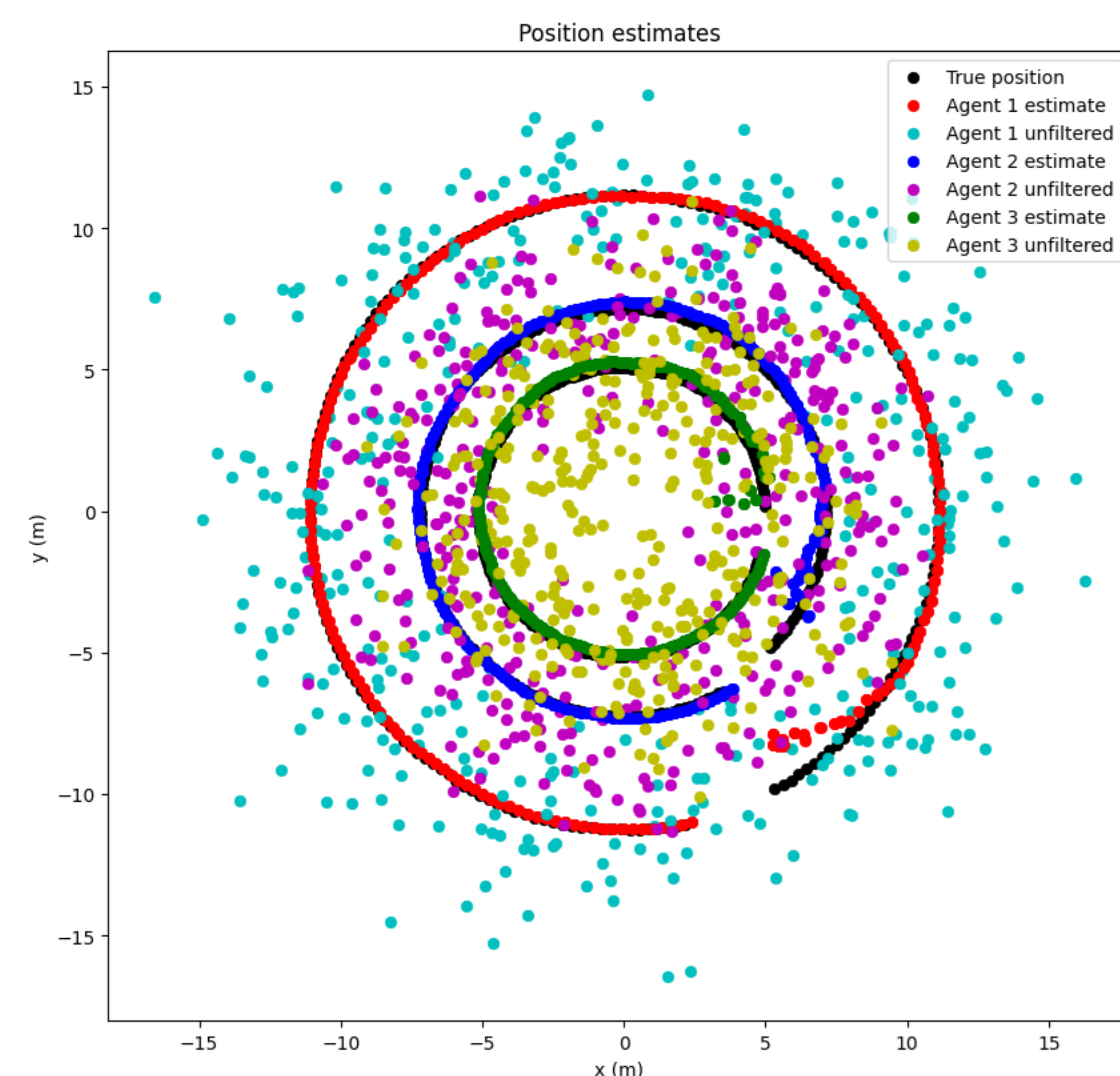


Figure 1. Simulation of distributed Kalman filtering algorithm on 3 mobile agents.

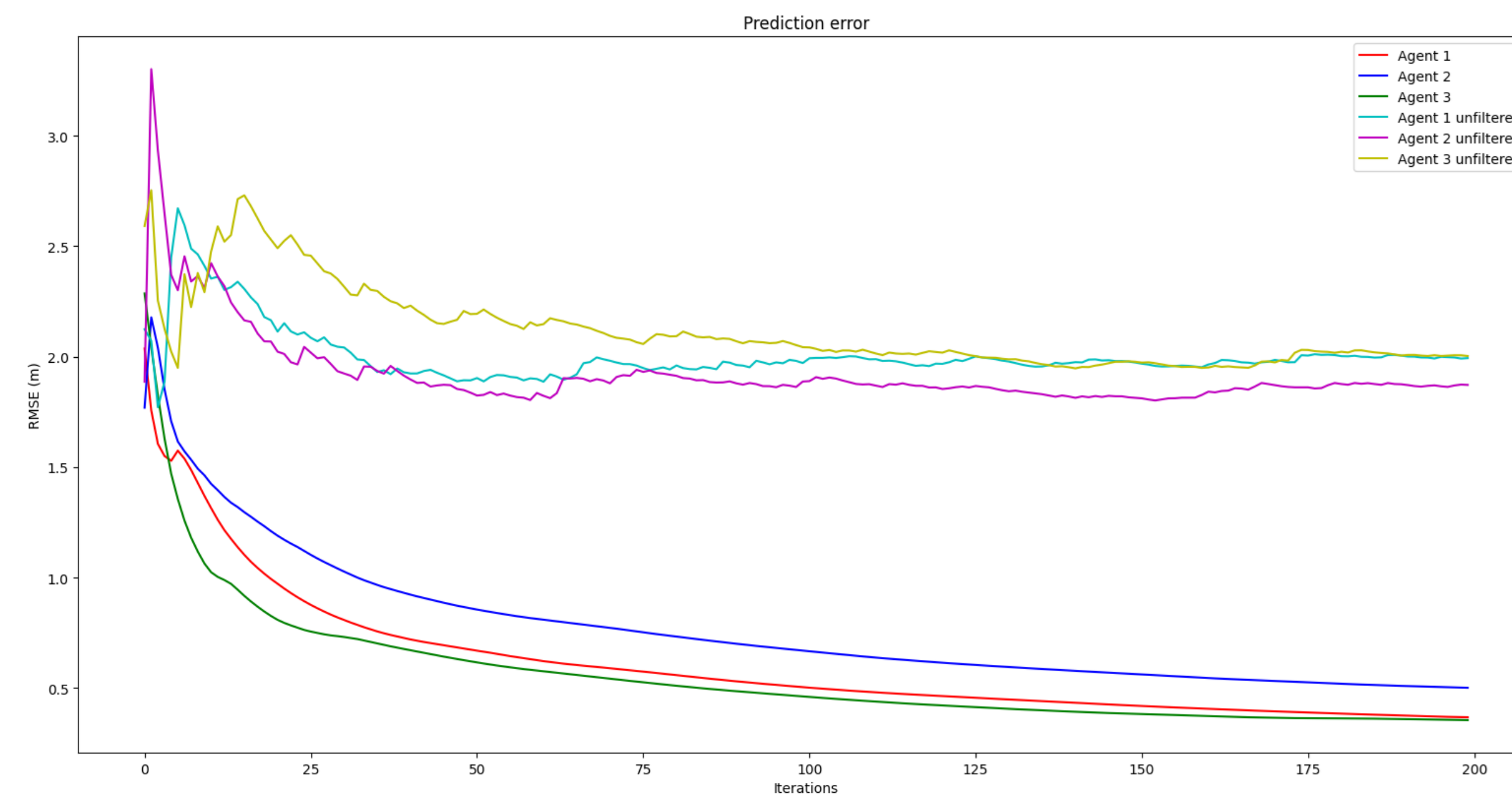


Figure 2. Root mean square error (RMSE) of simulated agent position estimates.

Iterations	5	10	20	50	100	150	200
Agent 1 RMSE (m)	1.528	1.370	0.994	0.674	0.504	0.420	0.368
Agent 2 RMSE (m)	1.707	1.463	1.210	0.861	0.669	0.563	0.501
Agent 3 RMSE (m)	1.470	1.064	0.826	0.621	0.462	0.383	0.355

(a)

	Percentage of iterations where RMSE > 1.0m
Agent 1	9.5%
Agent 2	16.0%
Agent 3	6.0%

(b)

Figure 3. Summarizing data table of Figure 2: (a) RMSE data and (b) additional characterization of algorithm performance.

	Measurement noise	System noise
Case 1	$R = 10 \cdot I_2$	$Q = 10 \cdot I_2$
Case 2	$R = 10 \cdot I_2$	$Q = 10 \cdot I_2$

Figure 4. Experimentally determined noise limits shown as covariance matrices

## Formation Control

### Motivation

- Most current formation control techniques for multi-agent systems are tedious and time consuming.
- Autonomous formation control based on localization is more flexible and robust.

### Background

- Network of n rigid bodies in 3D space
- Reynold's Flocking Principles: cohesion, separation, alignment
- Rigid body pose:

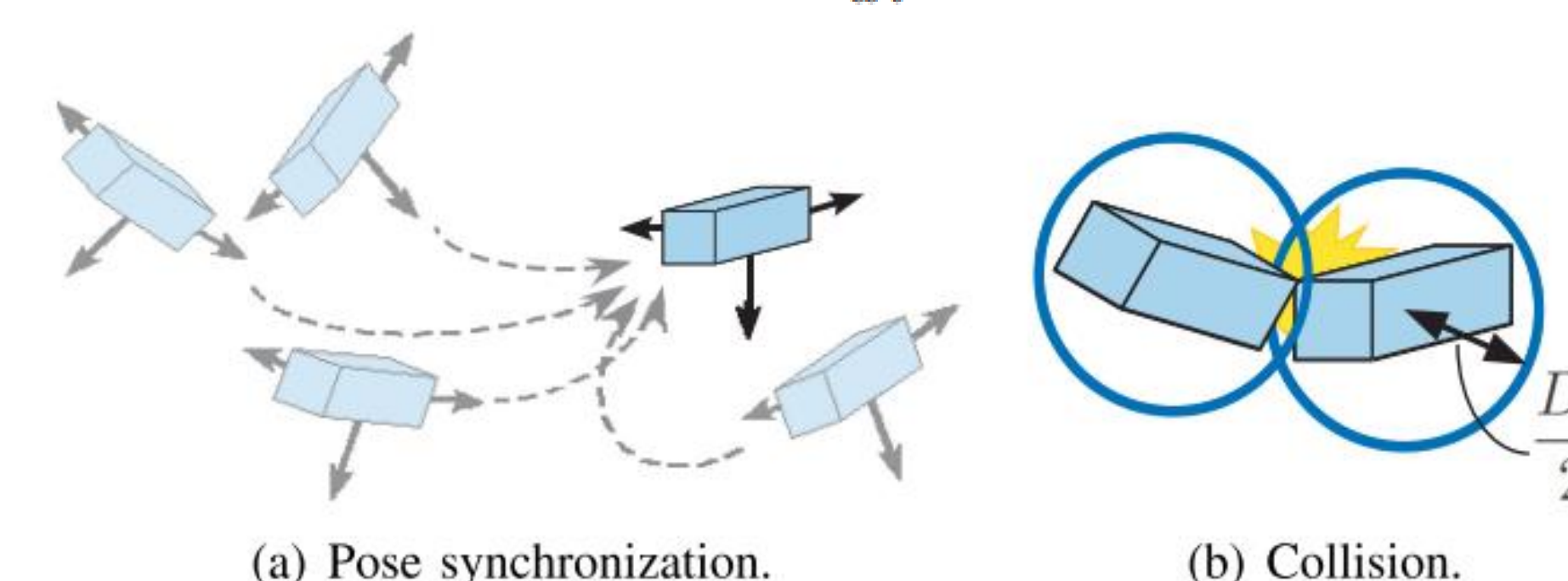
$$g_{ij} := g_{wi}^{-1} g_{wj} = (p_{ij}, e^{\epsilon \theta_{ij}}) \in SE(3)$$

- Rigid Body velocity:

$$V_{wi}^b = [(v_{wi}^b)^T (\omega_{wi}^b)^T]^T \in \mathbb{R}^6$$

- Rigid body motion dynamics:

$$\dot{g}_{wi} = g_{wi} \hat{V}_{wi}^b$$



### Methodology

- Distributed Control Approach:
  - Localization allows pose information to be derived from neighbors
  - Each agent adjusts itself in relevance to its neighbors
- Zeroing Control Barrier Function For Collision Avoidance:

$$p_{ij}^T v_{wi}^b \leq k_c (\|p_{ij}\|^2 - D_c^2) \quad \forall j \in \mathcal{N}_{di}, k_c > 0$$

- Conditions for Pose Synchronization:

$$\sum_{j \in \mathcal{N}_i} p_{ij}^T v_{wi}^b \geq k_p \left\| \sum_{j \in \mathcal{N}_i} p_{ij} \right\|^2 \quad \sum_{j \in \mathcal{N}_i} (\log(e^{\epsilon \theta_{ij}})^v)^T \omega_{wi}^b \geq k_e \left\| \sum_{j \in \mathcal{N}_i} \log(e^{\epsilon \theta_{ij}})^v \right\|^2$$

- Quadratic Problem Formulation: The Control Objective

$$V_i^* = \arg \min_{V_{wi}^b \in \mathbb{R}^6, \delta_i \geq 0} (V_{wi}^b)^T V_{wi}^b + \delta_i^2$$

- Theoretical Guarantees:
  - Pose synchronization achieved as much as possible with collision avoidance
  - Formation robust to swarm movement
- Extensions:
  - Flocking with desired behavior (e.g. moving towards a beacon)
  - Modification for 2D ground vehicles with nonholonomic constraints

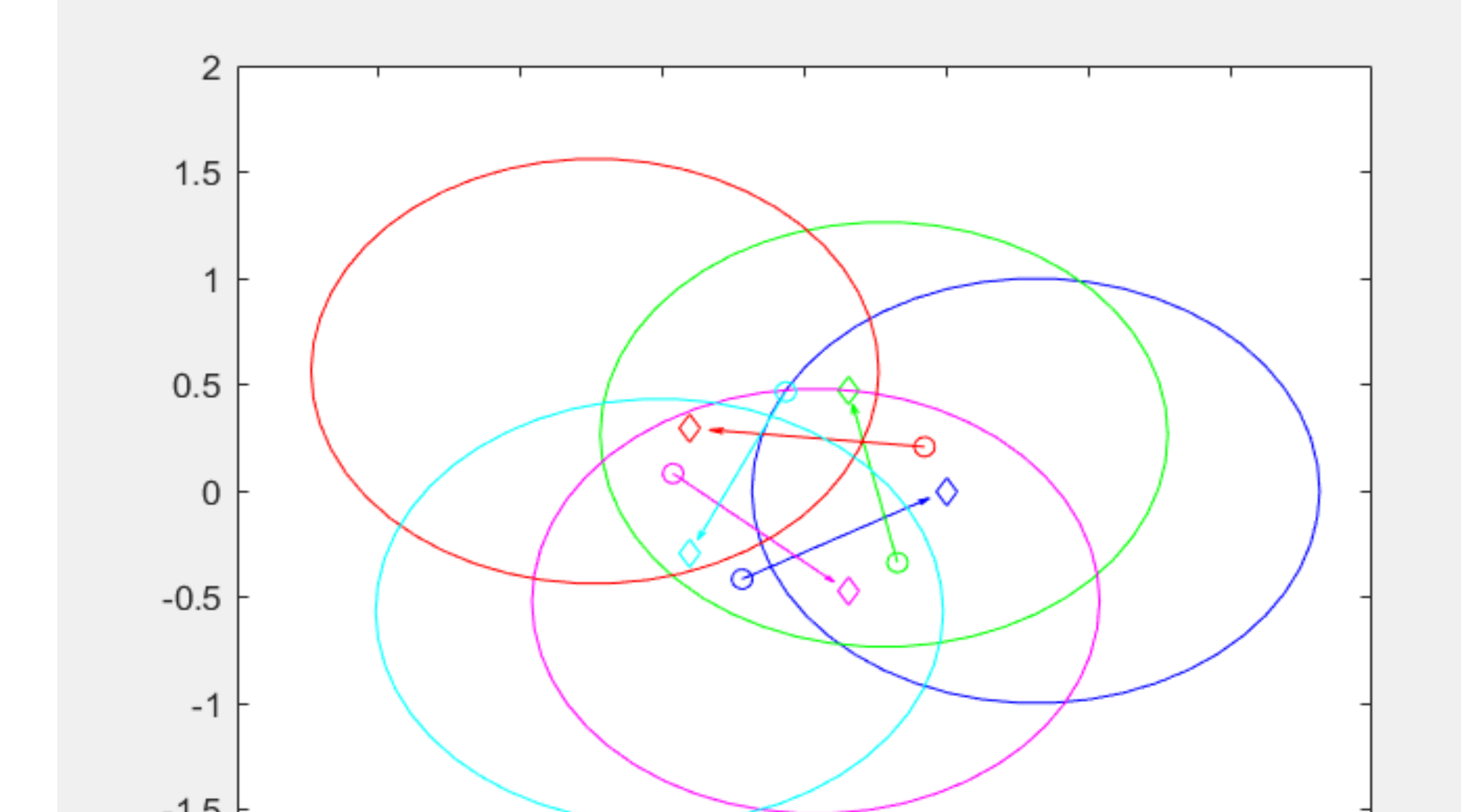


Figure 4: Circular formation with concentric initialization

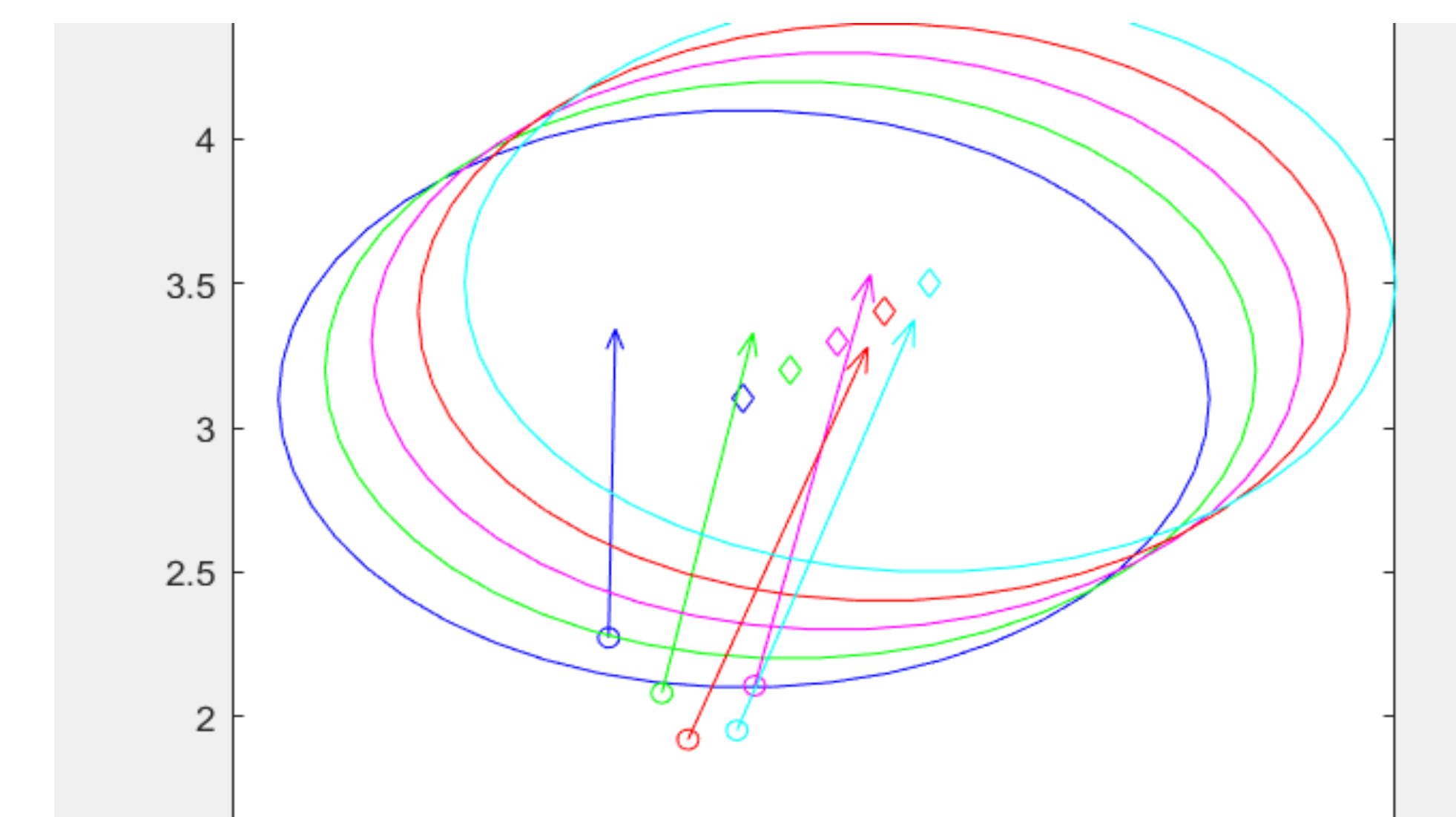


Figure 5: Linear formation with circular initialization some distance away.

## Future Work

- Address algorithm limitations
- ROS Humble and Gazebo simulations
- Turtlebot and UAV drone implementation
- Collision avoidance for environmental obstacles
- Agent loss independence and formation recovery
- Large-scale swarms
- Extension to swarm tasks
  - Follow target or beacon in formation
  - Environmental or object manipulation

### Acknowledgements

This research was funded by the Electrical Engineering Department at Cal Poly.

### References

- 1 R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," 2007 46th IEEE Conference on Decision and Control, New Orleans, LA, USA, 2007, pp. 5492-5498.
- 2 Ibuki, T., Wilson, S., Yamauchi, J., Fujita, M., & Egerstedt, M. (2020). Optimization-based distributed flocking control for multiple rigid bodies. IEEE Robotics and Automation Letters, 5(2), 1891-1898.
- 3 C. W. Reynolds, "Flocks, herds and schools: A distributed behavioral model," Comput. Graph., vol. 21, no. 4, pp. 25-34, Jul. 1987.